Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

other side of that diameter. If the number of sides of the polygon is uneven there will be no such symmetry.

Let MN be a diameter, $ABCDEFG$ be a regular polygon of n sides, n being an odd number, and O the center. Draw all the radii to the vertices, then $\angle ACB = \angle BOC = \angle COD = \ldots\ldots = 2\pi/n = \beta$. Denote $\angle AOM$ by a, $\angle GOM$ by a'. It will be perceived at once that on one side of the diameter MN there are $\frac12(n-3)$ triangles, and on the other $\frac12(n-1)$ triangles, not cut by the diameter.

Employing the well-known formula of the difference of two cosines, we have

$$\cos(a-\tfrac12\beta)-\cos(a-\tfrac12\beta)=2\sin a\sin\beta,$$

$$\cos(a+\tfrac12\beta)-\cos(a+\tfrac32\beta)=2\sin(a+\beta)\sin\tfrac12\beta,$$

$$\cos(a+\tfrac32\beta)-\cos(a+\tfrac52\beta)=2\sin(a+2\beta)\sin\tfrac12\beta$$

$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$

$$\cos[a+\tfrac12(2m-1)\beta]-\cos[a+\tfrac12(2m+1)\beta]=\pm2\sin(a+m\beta)\sin\tfrac12\beta.$$

Adding,

$$\frac{\sin(a+\frac12 m\beta)\sin\frac12(m+1)\beta}{\sin\frac14\beta}=\sin a+\sin(a+\beta)+\sin(a+2\beta)+\ldots\ldots+\sin(a+m\beta).$$

Putting the radius of the circle=1, this last formula gives us the sum of the perpendiculars drawn from the vertices upon MN on either side of the latter.

Putting $m=\frac12(n-3)\beta$, this sum

$$=\frac{\cos(a_1-\frac34\beta)\cos\frac14\beta}{\sin\frac12\beta}.$$

Putting $m=\frac12(n-1)$ we get

$$\frac{\cos(a_1-\frac14\beta)\cos\frac14\beta}{\sin\frac12\beta},$$

but $a_1=\beta-a$, therefore the latter expression is

$$\frac{\cos(\frac34\beta-a)\cos\frac14\beta}{\sin\frac12\beta}.$$

Consequently we have equality for the two sums of the perpendiculars, which proves the theorem.

Also solved by *G. B. M. ZERR*.

123. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Science, Decorah Institute, Decorah, Iowa.

A etant le point d' intersection des médianes d' un triangle ABC, démontrerque $AB^2+BC^2+CA^2=3(GA^2+GB^2+GB^2)$. [Ex. 84, Géométrie. No. 2, 1^{re} Anne *L' Éducation Mathématique.*]

I. Solution by **M. A. GRUBER, A. M.**, War Department, Washington, D. C.; **CHARLES C. CROSS**, Whaleyville, Va.; and **H. C. WHITAKER, Ph. D.**, Professor of Mathematics, Manual Training School, Philadelphia, Pa.

Take $BC=a$, $AC=b$, and $AB=c$, and put the medians equal, respectively, m_a, m_b, and m_c. Then

$$m_a=\tfrac{1}{2}\sqrt{[2(b^2+c^2)-a^2]},$$
$$m_b=\tfrac{1}{2}\sqrt{[2(a^2+c^2)-b^2]},$$
$$m_c=\tfrac{1}{2}\sqrt{[2(a^2+b^2)-c^2]}.$$

But the medians of a triangle intersect at a common point two-thirds of the distance from the vertex to the middle of the opposite side. Whence

$$GA^2=(\tfrac{2}{3}m_a)^2=\tfrac{2}{9}b^2+\tfrac{2}{9}c^2-\tfrac{1}{9}a^2,$$
$$GB^2=(\tfrac{2}{3}m_b)^2=\tfrac{2}{9}a^2+\tfrac{2}{9}c^2-\tfrac{1}{9}b^2,$$
$$GC^2=(\tfrac{2}{3}m_c)^2=\tfrac{2}{9}a^2+\tfrac{2}{9}b^2-\tfrac{1}{9}c^2.$$

$\therefore$ $GA^2+GB^2+GC^2=\tfrac{1}{3}(a^2+b^2+c^2)$, and $3(GA^2+GB^2+GC^2)=a^2+b^2+c^2=BC^2+AC^2+AB^2$.

II. Solution by **J. W. YOUNG**, Fellow and Assistant, Ohio State University, Columbus, O.; **P. S. BERG, B. Sc.**, Principal of Schools, Larimore, N. D.; and **J. SCHEFFER, A. M.**, Hagerstown, Md.

Let ABC be the triangle, AM, BN, and CL the medians, and G the intersection of the medians. Then by a well-known theorem,

$$AC^2+CB^2=2CL^2+2.AL^2=2CL^2+\tfrac{1}{2}AB^2.$$

Similarly,

$$CB^2+BA^2=2BN^2+\tfrac{1}{2}AB^2,$$
$$BA^2+AC^2=2AM^2+\tfrac{1}{2}CB^2.$$

Adding and dividing by 2, we have

$$AC^2+CB^2+BA^2=CL^2+BN^2+AM^2+\tfrac{1}{4}(AB^2+AC^2+CB^2),$$
$$\text{or } AC^2+CB^2+BA^2+\tfrac{4}{3}(CL^2+BN^2+AM^2),$$

(since G divides medians in the ratio 2:1)

$$=\tfrac{4}{3}(\tfrac{9}{4}CG^2+\tfrac{9}{4}BG^2+\tfrac{9}{4}AG^2)=3(CG^2+BG^2+AG^2.$$

The same propositions can very easily be proven analytically.

Solved in a similar manner by *COOPER D. SCHMITT*, *G. B. M. ZERR*, *WALTER H. DRANE*, and *CHAS. C. CROSS*.

124. Proposed by **B. F. FINKEL, A. M., M. Sc.**, Professor of Mathematics and Physics in Drury College, Springfield, Mo.

Every conic that passes through all the foci of a conic is a rectangular hyperbola. [From Charlotte A. Scott's *Modern Analytical Geometry.*]